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A Variable Neighborhood Search Algorithm for the Multi-stage Weapon Target Assignment Problem

Xuening Chang [1], Jianmai Shi [1+], Chao Chen [1]

[1] School of Systems Engineering, National University of Defense Technology

Abstract. The weapon target assignment (WTA) problem is an important research topic in military operations research and is known to be NP-complete. As a new variant of WTA, the multi-stage weapon target assignment (MS-WTA) problem is proposed for large scale military operations and is more complex. In this paper, the mathematical model of MS-WTA is developed, where the objective is to minimize the expected enemy residual value and the constraints of weapon and target in different stages are included. Based on the framework of variable neighborhood search algorithm, specific neighborhood search structures and shaking methods are designed and two decoding strategies are proposed. Furthermore, we compared the two strategies in MS-WTA though instances of different sizes. Experimental results show that the proposed algorithm is effective in solving small and medium-sized problems.

Keywords: WTA, variable neighborhood search algorithm, multi-stage;

1. Introduction

Weapon target assignment (WTA) problem is one of the most important issues in military operations research and application. The classical WTA problem refers to the allocation of a number of weapons to a set of enemy's targets, with the tactical intention of maximizing the overall damage of these targets. Either side of the war always seeks to destroy the enemy's important strategic resources and makes them lose their combat capability through scientific weapons allocation. The WTA problem has been proven to be NP-Complete [1]. Therefore, efficient heuristic and meta-heuristic algorithms for solving WTA have been widely studied.

In current wars, long-range precision-guided weapons acts more and more important roles for destroying targets. However, these weapons are quite expensive and their launching platforms are usually limited. When the number of targets is more than the number of the weapons' launching platforms. They usually have to consume multiple cycles to destroy the targets, which leads to the multi-stage weapon target assignment (MS-WTA) problem for better utilizing the utility of the long-range precision-guided weapons. The MS-WTA problem can be viewed as a new extension of WTA. Compared with general WTA, MS-WTA takes account to the time constraints of allocating weapons to the targets and the optimal attacking effects of the targets in different stages (time periods). The solution space of MS-WTA is more complicated and the optimal solution is more difficult to obtain. When the multi-stage situation is considered, the weapons usually cannot be used in all stages. The commander has to optimize the assignment of weapons to the targets as well as which stage the weapons are utilized.

Manne(1958) first investigated the WTA problem for defensing ballistic missiles [2]. Walkup(1964) used network graph approach for single stage WTA problem [3]. Eckler and Burr (1972) discussed the impact of probability uncertainties for destroying targets in two stages [4]. Then Chang et al. (1987) established a multi-stage model considering the value of targets in different stages [5]. Based on the former studies, Hosein (1988) proposed the concept of dynamic WTA (DWTA) [6]. In DWTA, the current assignment decision affects the firepower distribution scheme in the next stages [7]. Since then, mathematical models and relevant solution techniques for DWTA are further studied [8]-[10],[12]. Murphy (2000) proposed stochastic decomposition approximation technique to solve the damage uncertainty problem under multi-wave attack [11]. Inspired by Murphy's study, Ahner et al. (2015) proposed an adaptive

+ Corresponding author. Tel.: + 15388922730;
E-mail address: jianmaishi@nudt.edu.cn.

dynamic programming method to solve the two-stage WTA problem [7]. It's worth noting that Murphy and Ahner assume all weapons have the same kill probability to a target. Xin et al. (2011) relaxed the restriction on destroying probabilities, and developed a more realistic model on missile defense situation [12]. Then Davis et al. (2017) established a Markov decision process model of discrete wavelet transform to solve the DWTA problem [13]. Heuristic method is also an important method to solve DWTA or MS-WTA, genetic algorithm (GA) [14], particle swarm optimization (PSO)[15], and adaptive large neighbor search (ALNS)[16] are utilized.

As seen from current researches, more of works focus on single stage WTA and DWTA. Only a few of them studied some special and simplified multi-stage situations. It is of great significance to study MS-WTA as the increasing use of long-range precision-guided weapons and develop efficient algorithm for improving the assignment efficiency of these weapons.

The remainder of this paper is organized as follows. Section II presents the mathematical model of MS-WTA. Section III proposes the variable neighborhood search algorithm and describes the shaking and local search operators. In Section IV reports the computational experiments. Finally, section V discusses the conclusions and directions for future research.

2. Formulation

2.1. Problem description

We introduce a finite set of allocation weapons ***W***(denoted by i = 1, 2,…m), a finite set of stages ***S***(denoted by t = 1, 2,…, s) and a finite set of targets ***N***(denoted by j = 1, 2,…,n). The MS-WTA problem can be described as: the attacker intend to destroy n targets. According to the importance, the targets are divided into s stages(denoted by j $\in N_t$, $N_t \subseteq N$). Then all weapons also are divided into s stages (denoted by i $\in W_t, W_t \subseteq W$). For each *t*, allocate the weapon *i* to target *j,* wherein $i \in W_t$, j $\in N_t$ .Let x_{tij} be a binary variable that denote whether weapon *i* is assigned to target *j* in period *t*. Table Ⅰ describes relevant sets, parameters and variables.

Table 1: symbols description

Sets:	
W	a finite set of allocation weapons, $i \in W$
S	a finite set of stages, $t \in S$
N	a finite set of targets, $j \in N$
W_t	A subset of weapon in period t , $W_t \subseteq W$
N_t	A subset of target in period t , $N_t \subseteq N$
W_f	W_f ={1,2,…,f}, The total number of weapon launching platforms at each stage shall not exceed f
X	Three dimensional decision set, $x_{tij} \in X$
Parameters:	
p_{ij}	The probability of weapon i to target j
V_j	the destroy value of target j
D_j	damage threshold of the target j
m	the number of weapon types，which is equal to W
n	the number of target
s	the number of stage
f	The number of weapon launching platform

decision Variable:	
x_{tij}	the decision variable, which is a 0-1 binary number

Manne (1958) investigated the WTA problem for ballistic missile defense and developed a nonlinear programming model [2], which is used by many following studies such as Lemus and David (1963) [16], Lee et al (2002) [17][18], and Ahner and Parson (2015) [6]. This model is a classical model for the single period static WTA problem and is formulated as follows.

$$\min \sum_{j=1}^{n} V_j \prod_{i=1}^{m} \left(1-p_{ij}\right)^{x_{ij}} \quad (1)$$

In classical DWTA problem, the early decision solution affects the sequential assignment. Take the shoot-look-shoot scenario of DWTA for example, with the battle progressing, the number of targets is uncertain and the optimal solution is more difficult to obtain. In order to simplify the complexity of the research and distinguish it from DWTA problem, the MSWTA problem studied in this paper makes the following assumptions:

- Once the sensing system completes the detection, the targets will not escape or increase.
- One weapon can only attack one target, but one target can be attacked by multiple weapons, which is called multi-to-one (weapon-to-target) model.
- The target must be attacked in certain stage and only up to the damage threshold can the target been removed from unassigned-target list.

2.2. Mathematical formulation

According to assumption 2 above, we can transform (2.1) into a simple nonlinear objective function, as following (2.2) shows. In particularly, when the number of the same weapon exceeds one, it can be viewed as a new weapon with the same parameters. It simplifies the calculation process by increasing the number of decision variables to narrow the domain of decision variables. The formulation of MS-WTA problem in this paper is as follows:

Formulation 1

$$\min \sum_{j=1}^{n} V_j \prod_{i=1}^{m} \left(1-x_{tij} p_{ij}\right) \quad (2)$$

$$1-\prod_{i=1}^{m} x_{tij}\left(1-p_{ij}\right) \geq D_j, j=1,2,\cdots, \text{n}; \quad (3)$$

$$\sum_{j=1}^{n}\sum_{t=1}^{s} x_{tij} \leq 1, i=1,2,\ldots,m \quad (4)$$

$$\sum_{t}^{s}\sum_{i}^{m}\sum_{j}^{n} x_{tij} \leq m \quad (5)$$

$$\sum_{i\in W_t}\sum_{j\in N_t} x_{tij} \leq f, t=1,2,\cdots \quad (6)$$

$$x_{tij} = \begin{cases} 0 & weapon\ i\ is\ assigned\ to\ target\ j \\ 1 & otherwise \end{cases} \quad (7)$$

In **formulation 1**, (2) is the objective function, and (3) - (6) are constraints. Equation (2) represents that the overall residual value of all target is minimized. We hope to reduce the probability of high-value targets surviving in combat aim at achieving best strike effects. When the weapon inventory is limited, this objective function contains one hidden require: the commander have to choose more lethal weapons as much as possible for important and high-value targets. Equation (3) makes sure that the actual damage caused by all weapons assigned to a target must exceed the prescribed damage threshold; Equation (4) is the numerical representation of assumption 2, which imposes that a weapon can only be assigned to no more than one target; As a global constraint, equation (5) stipulates that when the allocation is completed, the total amount of weapons used cannot exceed the inventory of weapons; In order to fail to exceed the max ability of launching platform , (6) limited the number of available weapon in every stage. Equation (7) indicates that the decision variable is a binary number, which is equal to 1 when it is allocated, otherwise it is equal to 0.

3. Variable Neighborhood Search Algorithm

Variable neighborhood search (VNS) algorithm is an improved local search algorithm. The mechanism of alternate selection of multiple neighborhood structures enables the algorithm to explore solution space deeply, taking the ability of local optimization and global optimization into consideration. The framework of multi-stage WTA algorithm based on variable neighborhood search is as follows:

Algorithm 1: the structure of variable neighborhood search algorithm

***Input**:* a series of neighborhood structures like N_p , for $p = 1,2... , P_{\max}$ called search set.
a series of shaking structures like D_q ,for $q = 1,2...$, $q_{\max}$ called shaking set.
a initial solution s_0 ;global best solution s_{gb} ; local best solution s_{lb}
a initial target code list TP
***output**:* global best solution s_{gb} and a new target code list TP
Initialize all parameters
while stop == False:
global best solution $s_{gb} \leftarrow s_0$; local best solution $s_{lb} \leftarrow s_0$
pick up a shaking structures D_q randomly from shaking set.
get a local best s_{lb} by VNS
If $s_{lb} < s_{gb}$:
$s_{gb} \leftarrow s_{lb}$
Else:
$s_{gb} \leftarrow s_0$
$s_{lb} \leftarrow s_{lb}$
Until stopping criteria
Stop = true
End while
***output**:* global best solution s_{gb} and a new target code list TP

3.1. Multi-neighborhood Structure

In algorithm 1, we call the alternate search mechanism using different neighborhood actions multi-neighborhood structure (MNS). MNS contains a variety of neighborhood search operators, which will be introduced in section D. In this paper, the encoded list of the target is the operator's operation object, and a new solution is found by local search. If the quality of the new solution is worse than the current optimal solution, the next neighborhood operator is selected; If the new solution is better than the current solution, then we go to the first neighborhood operator to find the local optimal solution of the current solution. In MNS structure, the design of neighbor operators needs to keep a balance between "width" and "precision". When solving small-scale problems, we pay more attention to precision. Only when the precision is high can the algorithm find the optimal solution with greater probability. However, when the scale of the problem becomes larger, the high-precision local search will lead to unacceptable execution time, so we need to solve the problem by a coarser way to shorten the run time. An effective variable neighborhood search algorithm must be a combination of multiple neighborhood actions.

3.2. The Encoding Method

For the MSWTAP, it is simple to construct a three-dimensional array directly by binary coding, but the operation of local search is complex, and the algorithm needs to perform a large number of repair operations, which greatly prolongs the solution time. Therefore, we propose to use the target-based numerical coding method, which converts the original data information into multiple target lists by means of analysis, and then uses a limited number to correspond to the lists one by one. Its representation method is shown in Figure 1.

Stage→	1st-stage		2nd-stage		3rd-stage	
Target coding→	1	5	2	3	4	6
Final target coding→	0	1	2	3	4	5

Fig. 1: Encoding method target-based in algorithm.

3.3. Greedy Strategy Algorithm

(1) General greedy strategy algorithm (GGSA)

General greedy strategy algorithm (GGSA) is a simple constructive heuristic algorithm. In the process of constructing feasible solutions, each step only focuses on the most favorable combination for the current task. This algorithm is not only a decoding strategy, but also an important method to construct the initial solution in this paper. Compared with the completely random allocation method, the quality of the solution is higher, but the solution usually obtained is not the global optimal solution of the problem. Here, we use the algorithm based on greedy strategy as one of the methods to construct the initial feasible solution. The following is a schematic diagram of the algorithm:

Algorithm 2: General greedy strategy algorithm, GGSA

1 ***inputs:*** Parameter Initialization: weapon set W; target set N; fire probability matrix P
Target coding list TP; initial solution $X=[0]_{s\times m\times n}$
2 ***outputs:*** ini_sol $X=\left[x_{tij}\right]_{s\times m\times n}$
3 ***While*** N != False
4 select a target j by the numbers order
5 select feasible weapon i based on greedy strategy, that is $i=\text{argmax}\left\{p_{tij}\right\}$
6 calculate the destroy value d_{ij} caused by wepon i
7 if $\sum_i d_{ij} < D_j$, $W=W\setminus i$ and $x_{tij}=1$
8 repeat line 5 to line 7
9 else $W=W\setminus i$ and $N=N\setminus j$ and $x_{tij}=1$
10 ***end while***
11 ***output:*** $X=\left[x_{tij}\right]_{s\times m\times n}$

(2) greedy strategy algorithm with random factor (RGSA-VNS)

We find that using GGSA, the probability of program falling into local optimum is relatively high. In order to make the algorithm jump out of local optimum, we introduce random factor, and then combine algorithm 2 to construct initial solution to improve the global optimization ability of the algorithm. In addition, this algorithm can be used as a comparison method to prove the performance of VNS. Specifically, on the basis of algorithm 2, we assign weapons to each target in turn, and choose the weapons with a random way. The greater the damage probability of weapons, the higher the probability of being selected, so as to increase the ability of the algorithm to jump out of the local optimal solution. The algorithm framework can refer to algorithm 2.

3.4. Operators design

In VNS algorithm, we need to design search operators in two parts. The first part is to design the perturbation operator, the other part is the local search operator. In this paper, we proposed three kinds of shaking operators and four kinds of neighborhood search operators. What's more, more search operators are obtained by changing the internal parameters of the operator.

3.4.1 shaking structure

- *two swap operator:* The operator is a slightly shaking structure. When the initial target code list is passed to the shaking operator, two points in the array are randomly selected and the shaking operation can be completed by exchanging numbers.
- *reverse operator:* Different from point-exchange operator, reverse operator is a perturbation mode that changes the original sequence in a larger area, and the chosen length is determined by random numbers. This large change is helpful for jumping out of the multi-local optimal solution, but it is also easy to miss the global optimal solution.
- *Shuffle and recombination operator:* In order to achieve the balance between the ability to jump out of local optimum and the search accuracy, we design an recombination operator. Taking the target sequence as a piece of paper tape, we randomly cut off a piece with a fixed width, and shuffle the numerical order in this piece, finally recombine this piece with the remaining parts. This method can not only preserve the original structural features to a certain extent, but also disturb the current sequence violently.

3.4.2 Variable neighborhood structure

- *swap search operator:* Two-swap operation is a traditional local search method, which searches for the local optimal solution of a neighborhood structure by swapping a number and its neighbors with fixed intervals. In this algorithm, we use 2-interval, 3-interval and 4-interval search methods to find the local optimal solution.
- *reverse operator:* The reverse operator searches for local solutions by selecting two points with suitable spacing and then flipping all the numbers between the two points. In this algorithm, we design flip operators whose lengths are 0.1 times, 0.15 times and 0.2 times of the total target sequence length.
- *Crossover search operator:* The crossover search operator is divided into three steps: first, we divide the original sequence into four parts with equal length by quartering. Secondly, it traverses the points of the first quarter segment, intercepts the fixed-length segments in order, and performs the same operation at other three quarters. Finally, perform the crossover and reverse operation on the two fragments, the specific operation method is shown in Figure 2.

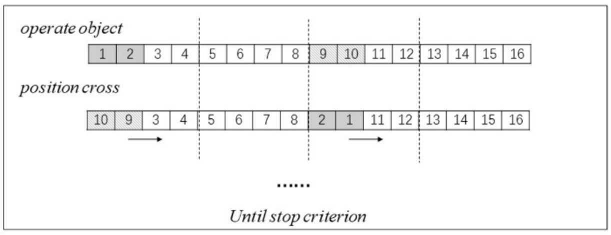

Fig. 2: crossover search operator

4. Computational Experiments

In order to verify the effectiveness of the VNS algorithm on MSWTAP, two sets of experiments in four scenarios are implemented in a computer having an Intel(R) Core(TM) i5-10210U CPU and 16GB RAM. The size of the instances ranges from 10 weapons and 5 targets to 60 weapons and 40 targets. The weapon and target data of each scenario comes from a random case generator. In order to avoid the influence of random factors on the experiment as much as possible, our algorithms are run 20 times, taking the average values of the loss function, and some experimental results are shown in Table 2. In order to further explore the change of VNS algorithm in running time, we counted the running time of RGSA-VNS algorithm in various scenarios, as shown in Table 3.

Table 3: running time

Problem Size	3*10*5	3*20*10	3*40*30	3*60*40
run time(/s)	0.257	0.743	59.938	249.76

According to the experimental results above, the two algorithms both can find the optimal solution of small-scale problems in a short time, but the initial solution quality of the variable neighborhood search algorithm with random factor (RGSA-VNS) is worse than that of the general variable neighborhood search algorithm (GGSA-VNS). The RGSA-VNS can achieve better results in finding the optimal solution of big size multi-stage problems, while the VNS algorithm based on pure greedy strategy can not explore the solution space adequately. Because it is easy to fall into local optimum. In addition, when the number of weapons is insufficient, the number of feasible solutions decreases sharply, the VNS algorithm always fail to complete strike task. In terms of running time in table 3 , the algorithm can solve the minimum scale problem in about 0.3s, but when the size of the weapon target rises to 60*40, on average, the algorithm will spend 249.76s, which means we need much more time to search a better solution when the size of problem is enlarged furtherly.

Table 2: The result of loss function value

Problem Size	GGSA-VNS		RGSA-VNS	
stage*weapon*target	Ini-sol	average	Ini-sol	average
3*10*5	2.686	2.686	2.896	2.686
3*20*10	2.253	2.253	3.525	2.253
3*40*30	6.339	4.822	8.105	4.822
3*60*40	10.455	7.633	10.527	7.407

5. Conclusion

MS-WTA is a critical military operations research problem for modern wars. Through analyzing the constraints and characteristics of the multi-stage problem, this paper transforms multi-to-multi WTA problem into the multi-to-one, we develop an integer nonlinear programming model of MS-WTA. A variable neighborhood search algorithm based on greedy strategy with random factor is designed for efficiently solving the problem. The effectiveness of the method is tested through a series of simulation experiments. The MS-WTA is a relative new topic in the military operations research field, there are still many extensions that can be investigated in future. For more complex multi-stage situation, new and different models are required. Other algorithms, e.g simulated annealing algorithm, adaptive large neighborhood search algorithm and reinforcement learning, can also be introduced and improved to solve different extensions of MS-WTA.

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